The Dynamic Behaviour of Functionally Graded Material Plates Using Different Homogenization Schemes

Hari Krishnan K P, Shashi Dharan

Abstract— Functionally graded materials belong to a class of advanced materials characterized by the variation in properties as the dimension varies (usually along the thickness). The overall properties of FGM can be varied according to our needs, thus one of the main advantages of such a material is that it can be tailored specifically for serving a particular function that makes it unique from any of the base materials used in its synthesis. In the present project, a study on the dynamic behaviour is developed using Reddy's Third Order Shear Deformation Theory. The results thus obtained are compared with other standard results and thus validated. Also, the effect of various homogenisation schemes on the dynamic behaviour and its comparison with available results is carried out.

Index Terms— Bending analysis, Buckling analysis, Functionally graded materials, Homogenisation schemes, Natural frequency, Plates, Shear deformation theories, Transverse shear, Vibration.

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1 Introduction

Functionally graded materials (FGM) are those materials that are synthesised for a specific purpose by the gradual mixing of two or more different materials so that the properties of each of the base materials can be used according to the external environment. Unlike laminated composite materials, the variation of properties is as smooth as possible thus avoiding the phenomena of stress concentration which could lead to the delamination at the interface. They are thus highly anisotropic materials engineered with great precision in gradients of composition and structure to adapt to various purposes and to have definite properties in preferred orientation. Various mechanical properties like Poisson's ratio, Young's modulus, shear modulus and material density can be varied in any preferred direction using different laws for spatial distribution. The concept of FGM was first introduced in Japan in 1984 during a space plane project. Where a combination of materials was required that would form a thermal barrier capable of withstanding a surface temperature of about 2000 K and a temperature gradient of the order 1000 K across a 10-mm section. Recently this topic has gained much attention is European countries.

There is a vast potential for the application of such a material as we can control the material properties and thus build a material according to our requirement. These materials find their applications in fields of aerospace, medicine, defense, energy, optoelectronics etc. thus because of this the area of analysis of such material has attracted a lot of attention of many researchers in recent time.

*Kerala, India. E-mail: shashisubha@gmail.com*Concerning to the analysis and applications of such materials, one can find several published works focused on FGM structures. Reddy presented a theoretical formulation based on Navier's solutions of rectangular plates, and on third-order shear deformation theory (TSDT) to analyze through-thickness functionally graded plates. A dual-phase material was assumed to be isotropic, having a distribution that varies through-the-thickness according to the exponent power law.

In 1998, J. N. Reddy & C. D. Chin had done the thermomechanical analysis of functionally graded cylinders and plates. The dynamic thermo-elastic response of functionally graded cylinders and plates is studied. Thermomechanical coupling is included in the formulation, and a finite element model of the formulation is developed. The heat conduction and the thermos-elastic equations are solved for a functionally graded axisymmetric cylinder subjected to thermal loading. In addition, a thermos-elastic boundary value problem using the first-order shear deformation plate theory (FSDT) that accounts for the transverse shear strains and the rotations, coupled with a three-dimensional heat conduction equation, is formulated for a functionally graded plate. Both problems are studied by varying the volume fraction of a ceramic and a metal using a power law distribution. In 2004, Tongsuk and Nukulchai, studied some elasticity problems based on a Moving Kriging (MK) interpolation in the construction of shape functions for the element-free Galerkin method. They mentioned that the key of the MK interpolation is its interpolation property that allows for exact imposition of essential boundary conditions, similar to the conventional FEM. A similar work was done by Chen and Liew and Regarding meshless methods, it is important to highlight the work presented by Ferreira et al. in which it is proposed the use of multi-quadric radial basis functions (RBF) to study the deformations of a simply supported functionally graded plates modelled by the TSDT. In 2006, Serge Abrate considered the problems of free vibrations, buckling, and static deflections of FGM plates in which material properties varied through the thickness and showed

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that, all other parameters remaining the same, the natural frequencies of functionally graded plates are proportional to those of homogeneous isotropic plates and that the proportionality constant can be easily be predicted. In 2007, Hiroyuki Matsunaga conducted a Free vibration and stability of functionally graded plates according to a 2-D higher-order deformation theory where the material property was modelled according to the power-law distribution in terms of the volume fractions of the constituents. Also in recent studies done by Kennedy, the FG plates whose material properties varies continuously through there thickness are modelled as exactly equivalent plates composed of up to four isotropic layers each model based on CLPT, FSDT and HSDT respectively. In 2012, Loja et al. studied the influence of using different homogenisation schemes, namely the schemes due to Voigt, Hashin–Shtrikman and Mori–Tanaka, on the prediction of bounds for the average material properties of FGM particulate composite structures. In 2015, G.M.S. Bernardo et al. performed a study on the structural behavior of FGM plates static and free vibrations analyses by considering a continuous variation of their phases and thus of their properties, and by considering a discrete stacking of a sufficient number of layers, in order to ensure a less abrupt variation profile of their properties based either on a meshless method or on different approaches based on the finite element method. A comparative study of the performance and adequacy of the developed models is carried out through a set of illustrative cases focused on the study of static and free vibrations behavior of plate structures. In the present study, the dynamic behavior of FGM plates has been conducted and compared that is modelled using different homogenization schemes namely Voigt Scheme, Mori-Tanaka Scheme, Sigmoid Function and Exponential Function respectively. Its dynamic behavior has been studied for different aspect ratios (a/b) from 0.5 to 3, side-thickness ratio (a/h) from 1 to 20 and power exponent values from 0 to infinity. All calculations has been done based on J.N.Reddy's Third Order Shear Deformation Theory.

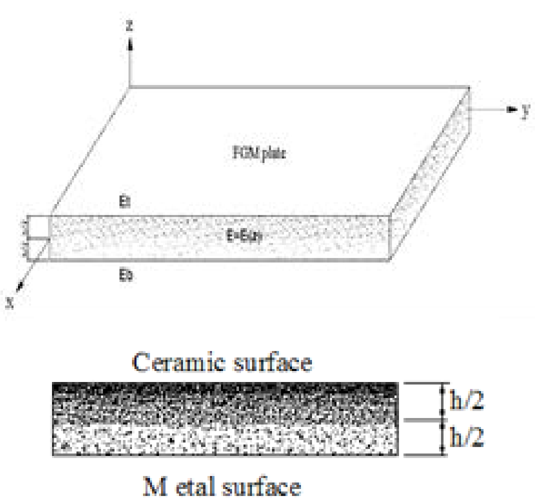

Fig.1. Schematic representation of a typical FGM plate.

2 Material Properties Gradation

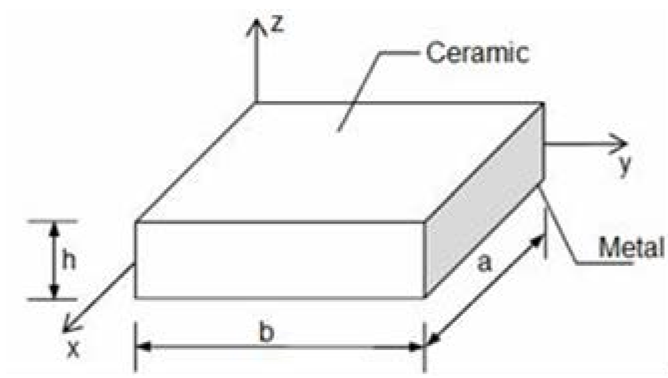

Fig.2. FGM geometry

The material properties gradation in FGM is assumed to follow power law function, exponential function etc.

1. **Exponential law**: This law is generally adopted when we deal with the fracture mechanics problems. According to this law the material property in P(z) in a specific direction is given by,

$$P(z) = P_t e^{\left(\frac{1}{h}\right)\left(\ln \frac{P_b}{P_t}\right)\left(z+\frac{h}{2}\right)}$$

Suffix 't' and 'b' re-presents the top and bottom surface of the plate respectively, 'h' is the thickness of the plate and 'z' is the specific location along the thickness direction.

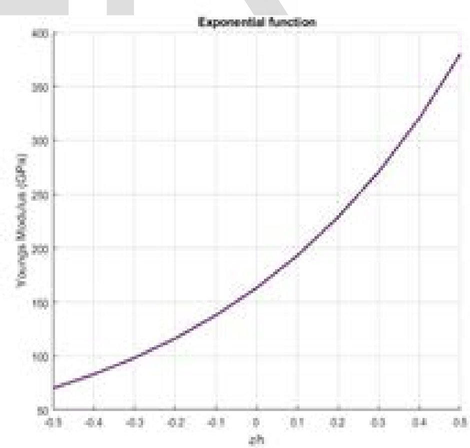

Fig.3. Variation of Young's Modulus with respt to non-dimensional thickness parameter according to Exponential Function

2. **Power Law**: It is observed from the open literature that this particular power law behavior is most used by many researchers. If FGM plate of uniform thickness 'h' is used for the analysis then according to this law, the material properties P(z) in a specific direction (along 'z') can be determined by,

$$P(z) = (P_t - P_b)V_f + P_b$$

It is noted that material properties are dependent on the volume fraction 'Vf' of FGM which follows the power-law as,

$$V_f = (z/h + 1/2)^n$$

where 'n' is the volume fraction exponent. Suffix 't' and 'b' re-presents the top and bottom surface of the plate respectively. The power law exponent 'n' can vary from '0' to '∞' that show the transition of material from fully ceramic to metallic phase, respectively.

3. **Sigmoid Function**: Power-law function and exponential function are commonly used to describe the gradation of material properties of FGMs but in both functions, the stress concentrations appear in one of the interfaces in which the material is continuous but changing rapidly. To overcome this, Chung and Chi, in their work suggested the use of another law called sigmoid law which is the combination of two power-law functions. This law in not independent law, it consists of two symmetric FGM layers having power-law distribution.
They also suggested that by the use of a sigmoid law the stress intensity factors of a cracked body can be reduced to a certain extend. According to this law, the two power-law functions are defined by,

$$f_1(z) = 1 - (0.5)\left(\frac{\frac{h}{2} - z}{\frac{h}{2}}\right)^p \quad 0 \le z \le \frac{h}{2}$$

$$f_2(z) = 1 - (0.5)\left(\frac{\frac{h}{2} + z}{\frac{h}{2}}\right)^p \quad -\frac{h}{2} \le z \le 0$$

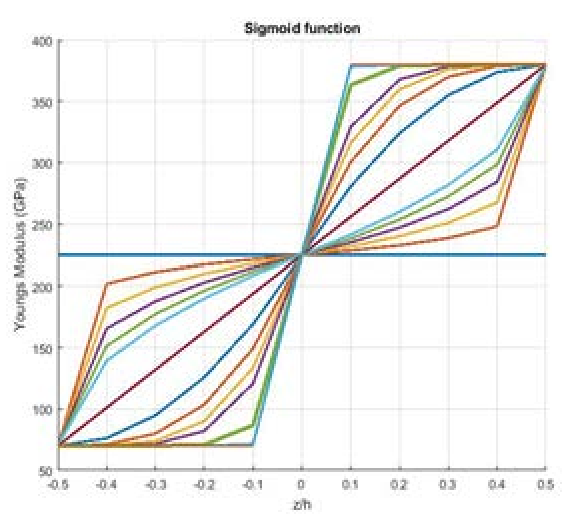

Fig.4. Variation of Young's Modulus with respt to non-dimensional thickness parameter according to Sigmoid Function

3 Effective Material Properties (Homogenisation) of FGM

The effective properties of macroscopic homogeneous composite materials can be derived from the microscopic heterogeneous material structures using homogenization techniques. Several models like rules of mixture (Voigt Scheme), Hashin–Shtrikman type bounds, Mori–Tanaka type models, and self-consistent schemes are available in literature for determination of the bounds of the effective properties. Voigt scheme and Mori–Tanaka schemes are generally adopted in analysis of functionally graded material plate and structure by most researchers.
Various methods to determine the effective properties of the plate are:

1. **Rule of mixtures:** It is similar to the power law as discussed in the previous section.

$$P(z) = (P_t - P_b)V_f + P_b$$

2. **Mori-Tanaka Scheme**: This method method works well for composites with regions of the graded microstructure have a clearly defined continuous matrix and a discontinuous particulate phase. The matrix phase is assumed to be reinforced by spherical particles of a particulate phase. The subscript 'e' denoted the effective value of a particular material property where as 'c' and 'm' denoted that of ceramic and metallic constituents respectively. The Bulk modulus(K) and Shear modulus (μ) is calculated as given below:

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m \frac{K_c - K_m}{K_m + 4/3\mu_m}}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m \frac{\mu_c - \mu_m}{\mu_m + f_1}}$$

Where,

$$f_1 = \frac{\mu_m(9K_m + 8\mu_m)}{6(K_m + 2\mu_m)}$$

Young's modulus (E), Poisson's ratio (υ) can be calculated from them as given below.

$$E = \frac{9K_e\upsilon_e}{3K_e + \upsilon_e}, \quad \upsilon = \frac{3K_e - 2\upsilon_e}{2(3K_e + \upsilon_e)}$$

3. **Voigt Scheme**: Voigt model has been adopted in most analyses of FGM structures. The advantage of Voigt method is that it is easy to calculate and can be considered as the upper and lower bounds for the effective elastic properties of a heterogeneous material. The effective material properties Pf, like Young's modulus Ef, Poisson' ra-

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tio vf, thermal expansion coefficient αf, and thermal conductivity Kf may be expressed as,

$$P_f = P_t V_c + P_b V_m$$

where Pt and Pb denoted the temperature-dependent proper ties of the top and bottom surfaces of the plate, respectively. Vm and Vc and are the metal and ceramic volume fractions which can be expressed by,

$$V_c + V_m = 1$$

If volume fraction Vm is assumed to follow a simple power law as

$$V_m = \left(\frac{2Z+h}{2h}\right)^n$$

where 'n' is the volume fraction index and takes only positive values then different effective properties can be given as,

$$E_f(Z,T) = [E_b(T) - E_t(T)]\left(\frac{2Z+h}{2h}\right)^n + E_t(T)$$

$$\alpha_f(Z,T) = [\alpha_b(T) - \alpha_t(T)]\left(\frac{2Z+h}{2h}\right)^n + \alpha_t(T)$$

$$K_f(Z,T) = [K_b(T) - K_t(T)]\left(\frac{2Z+h}{2h}\right)^n + K_t(T)$$

$$\nu_f(Z,T) = [\nu_b(T) - \nu_t(T)]\left(\frac{2Z+h}{2h}\right)^n + \nu_t(T)$$

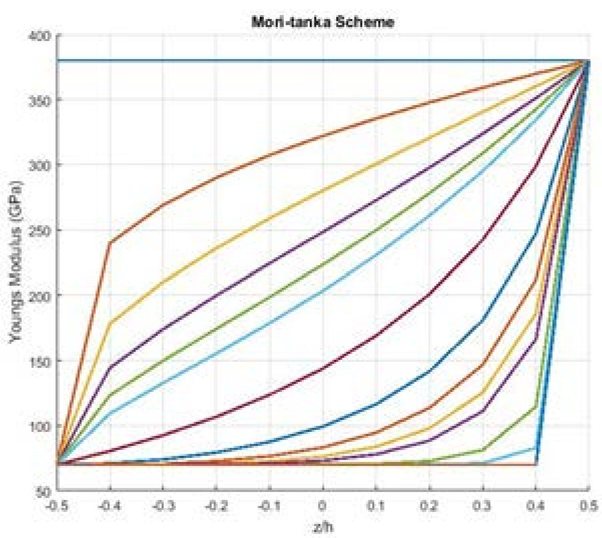

Fig.5. Variation of Young's Modulus with respt to non-dimensional thickness parameter according to Mori-Tanaka Scheme

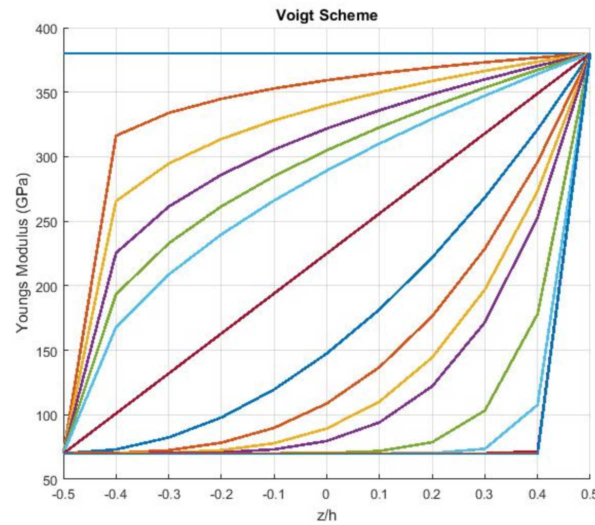

Fig.6. Variation of Young's Modulus with respt to non-dimensional thickness parameter according to Voigt Scheme

4 Plate Kinematics

The analysis of composite structures is one of the most promising research fields of the last decades. Accurate structural and dynamic analyses are required to design various structural parts of aerospace, mechanical, naval as well as civil constructions to find the behavior of the structural response in real time. Numerous plate theories have been developed by the researchers to analyze the composite plates and shells and out of them, the most commonly used ones are given below.
One of the major classification is the classical plate theory and shear deformation theories. Transverse shear stress components are neglected in the classical plate theory whereas it is included in the shear deformation theories.

4.1 Third Order Shear Deformation Theory(TSDT)

The free vibration analysis and stability analysis has been carried out using Reddy's Third Order Shear Deformation Theory. The displacement function used are as given below:

$$u = u_0 + z\phi_x - z^2\left(\frac{1}{2}\frac{\partial\phi z}{\partial x}\right) - z^3\left[C_1\left(\frac{\partial w_0}{\partial x} + \phi_x\right) + \frac{1}{3}\frac{\partial\phi z}{\partial x}\right]$$
$$v = v_0 + z\phi_y - z^2\left(\frac{1}{2}\frac{\partial\phi z}{\partial x}\right) - z^3\left[C_1\left(\frac{\partial w_0}{\partial y} + \phi_y\right) + \frac{1}{3}\frac{\partial\phi z}{\partial y}\right]$$
$$w = w_0 + z\phi_x + z^2\phi_z$$
$$C_1 = \frac{4}{3h^2},\ u_0 = u(x,y,0,t),\ v_0 = v(x,y,0,t),\ w_0 = w(x,y,0,t)$$

where,
'u', 'v', 'w' denotes the displacement variables. 'u0', 'v0', 'w0' are the in-plane displacements with respect to a reference plane,'w0' is the out-of-plane displacements with respect to the reference plane. 'ϕx', 'ϕy', 'ɵx', 'ɵy' are the rotation of normal with respect to mid-surface of the plate.

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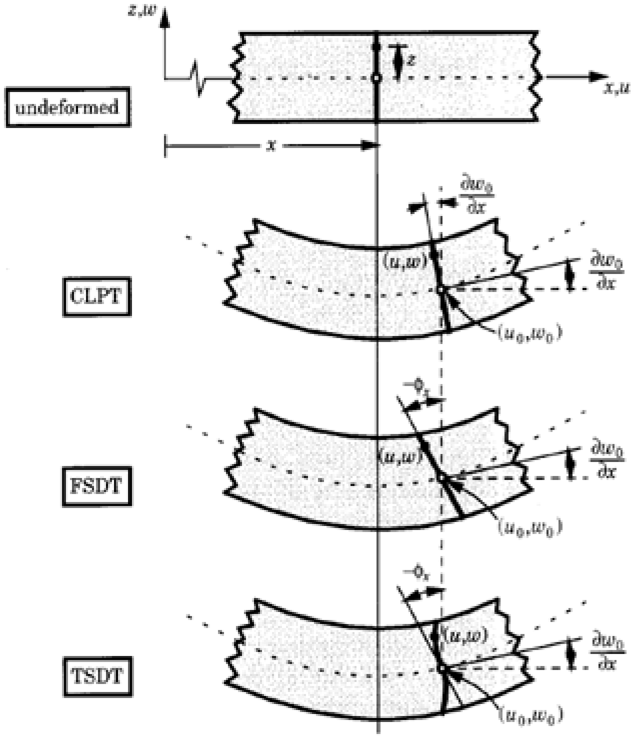

Fig.7. Tranverse shear deformation of a plane according to various plate theories

5 ANALYSIS

Analysis was done to calculate natural frequencies and buckling loads for different 'a/h', 'a/b' and 'n' values. This can be done based on Reddy's TSDT for different homogenization schemes. The Poisson's ratio can be considered constant or varying. Using TSDT we can generate stiffness matrix and mass matrix for natural frequency calculation. And we can assume a geometric stiffness matrix for calculation of buckling load.

5.1 Natural Frequency

For free vibration problems, the equations of motion can be expressed as the following eigenvalue problem:

$$([K] - \Omega^2 [M]) \{U\} = \{0\}.$$

where matrix [K] denotes the stiffness matrix which may contain the terms of the in-plane stresses and matrix [M], the mass matrix and {U} is the displacement vector corresponding to each node within the plate.

The square root of lowest value is taken as the natural frequency in each case.

5.2 Buckling Load

For stability problems, the natural frequency vanishes and the stability equation can be expressed as the following eigenvalue problem:

$$([K] - N [kg]) \{U\} = \{0\}.$$

where matrix [K] denotes the stiffness matrix and matrix [kg], the geometric-stiffness matrix due to the in-plane stresses. The lowest value is taken as critical buckling load.

6 VALIDATION

A validation study has been done for both natural frequency and buckling loads with available literature. The ceramic-metal material comprises of Alumina and Aluminiium. The material properties are given in table 1.

Table.1. Material properties considered.

Type	Name	Young's Modulus E (GPa)	Poisson's ratio, **υ**	Mass Density, **ρ** (Kg/m3)
Ceramic	Alumina	380	0.3	3800
Metal	Aluminium	70	0.3	2707

6.1 Natural Frequency

The natural frequency obtained using Reddy's TSDT has been validated. The result is made dimension-less using following expression,

$$\Omega = 10{*}\omega{*}h{*}\sqrt{\frac{\rho c}{Ec}}$$

where 'Ω' is the dimension-less natural frequency, 'ω' is the natural frequency, 'h' is the plate thickness, 'ρc' the mass density and 'Ec' is the young's modulus of the ceramic material.

Table.2. Natural frequency validation for different 'n' and 'a/h' values for square (Al2O3/Al) FGM plate.

a/h	Voigt Scheme							
	Power Exponent, n							
	0		0.5		1		4	
	(1)	[PW]	(1)	[PW]	(1)	[PW]	(1)	[PW]
5	2.11	1.98	1.805	1.78	1.631	1.627	1.4	1.3
10	0.577	0.54	0.49	0.483	0.441	0.441	0.382	0.36
20	0.148	0.138	0.125	0.124	0.113	0.113	0.098	0.093

[PW] -Present Work, (1) - G.M.S. Bernardo et al. (2015) [3]

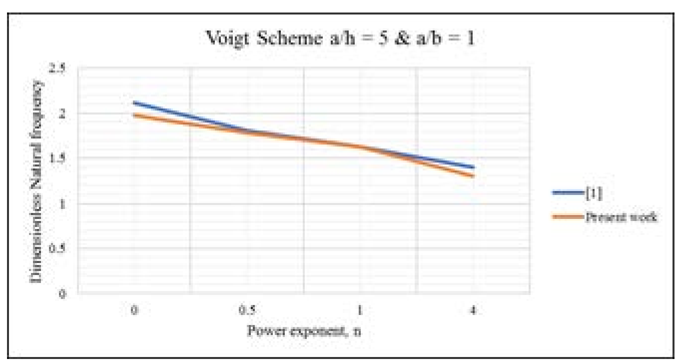

Fig.8. Variation of naural frequency w.r.t different 'n' values.

6.2 Buckling Load

The buckling load obtained using Reddy's TSDT has been validated. The result is made dimension-less using following expression,

$$N = \frac{Ncr}{h * Ec}$$

where 'N' is the dimension-less natural frequency, 'Ncr' is the natural frequency, 'h' is the plate thickness and 'Ec' is the young's modulus of the ceramic material.

Table.3. Uniaxial Buckling Load validation for different 'n' and 'a/h' values for square (Al2O3/Al) FGM plate.

Voigt Scheme				
n	[Work]	a/h		
		2	5	10
0	(1)	0.36	0.114	0.0338
	[PW]	0.32	0.118	0.034
0.5	(1)	0.25	0.0757	0.0221
	[PW]	0.23	0.0781	0.0223
1	(1)	0.19	0.0583	0.017
	[PW]	0.18	0.06	0.0171
4	(1)	0.1	0.0372	0.0113
	[PW]	0.1	0.0386	0.01138
10	(1)	0.09	0.0318	0.0099
	[PW]	0.07	0..0326	0.00994

[PW] -Present Work,
(1)- H. Matsunaga. et al (2008) [15]

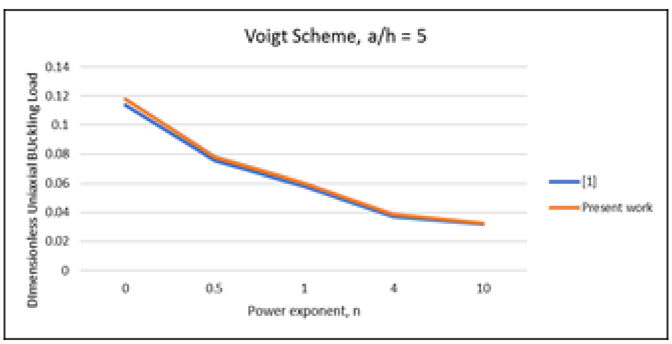

Fig.9. Variation of uniaxial buckling load w.r.t different 'n' values.

7 Comparative Study

A comparative study has been carried out for the results obtained for the FGM plate made of same materials but modelled by different homogenization schemes for different power exponent values, 'n' from 0 to infinity, side-thickness ratio, 'a/h' from 1 to 20 and aspect ratio, 'a/b' ranging from 0.5 to 3 respectively.
Materials given in table 1 is used for the comparative study.
The natural frequency, uniaxial and biaxial buckling loads obtained for different FGM plate models are considered for the study. The FGM plate modelled using Voigt Scheme, Mori-Tanaka Scheme, Sigmoid function and Exponential Function are named as V-FGM, M-FGM, S-FGM and E-FGM respectively. The top surface is ceramic rich and the bottom surface is metal rich. The plate is of uniform thickness, and simply supported on all four edges.

7.1 Natural Frequency

The natural frequency obtained using Reddy's TSDT has been validated. The result is made dimension-less using following expression,

$$\Omega = 10 * \omega * h * \sqrt{\frac{\rho c}{Ec}}$$

where 'Ω' is the dimension-less natural frequency, 'ω' is the natural frequency, 'h' is the plate thickness, 'ρc' the mass density and 'Ec' is the young's modulus of the ceramic material.

Table.4. dimensionless natural frequency for different power exponent 'n' for different homogenization schemes.

n	V-FGM	M-FGM	S-FGM	E-FGM
0	0.1975	0.1975	0.1760	0.1423
0.5	0.1780	0.1550	0.1698	0.1423
1	0.1628	0.1438	0.1630	0.1423
5	0.1278	0.1200	0.1480	0.1423
10	0.1207	0.1140	0.1460	0.1423
50	0.1070	0.1030	0.1447	0.1423
∞	0.0980	0.0980	0.1447	0.1423

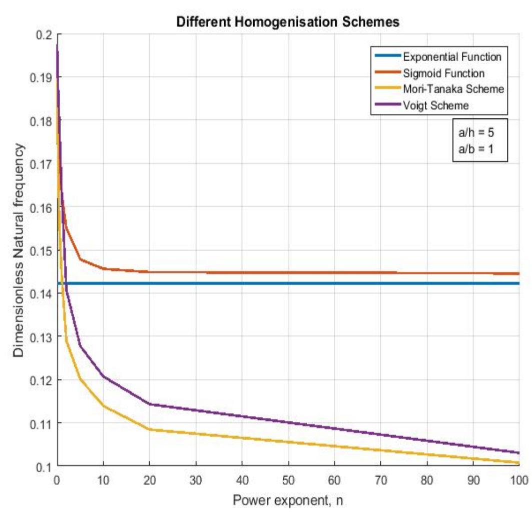

Fig.10. Variation of dimensionless natural frequency w.r.t 'n' values for a square FGM plate.

Table.5. dimensionless natural frequency for different aspect ratios 'a/b' for different homogenization schemes.

a/b	V-FGM	M-FGM	S-FGM	E-FGM
0.5	0.0916	0.0844	0.1004	0.0926
1	0.1407	0.1290	0.1549	0.1423
1.5	0.2150	0.196	0.2385	0.2179
2	0.3090	0.2795	0.3440	0.3124
2.5	0.4150	0.3735	0.4653	0.4200
3	0.5306	0.4746	0.5976	0.5367

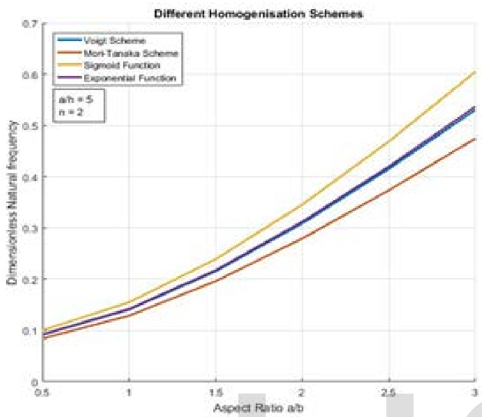

Fig.11. Variation of dimensionless natural frequency w.r.t 'a/b' values.

Table.6. dimensionless natural frequency for different side-thickness ratios 'a/h' for different homogenization schemes.

a/h	V-FGM	M-FGM	S-FGM	E-FGM
1	1.5832	1.3812	1.8244	1.5965
2	0.7070	0.7780	0.7069	0.6324
5	0.1407	0.1290	0.1549	0.1423
10	0.0384	0.0355	0.0418	0.0388
20	0.0098	0.0091	0.0107	0.0099

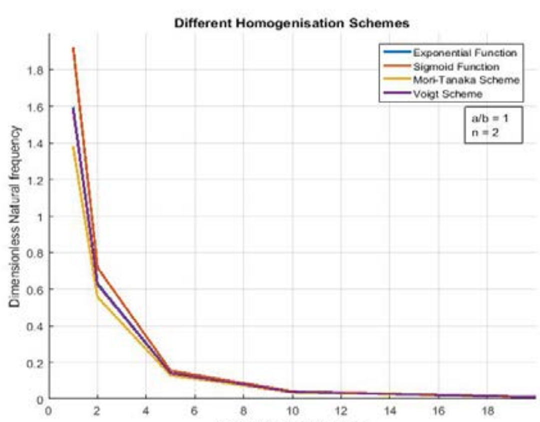

Fig.12. Variation of dimensionless natural frequency w.r.t 'a/h' values.

7.2 Uniaxial Buckling Load

The buckling load obtained using Reddy's TSDT has been validated. The result is made dimension-less using following expression,

$$N = \frac{Ncr}{h * Ec}$$

where 'N' is the dimension-less natural frequency, 'Ncr' is the natural frequency, 'h' is the plate thickness and 'Ec' is the young's modulus of the ceramic material.

Table.7. dimensionless uniaxial buckling load for different power exponent 'n' for different homogenization schemes.

n	V-FGM	M-FGM	S-FGM	E-FGM
0	0.1180	0.1180	0.0698	0.0459
0.5	0.0783	0.0596	0.0654	0.0459
1	0.0606	0.0473	0.0606	0.0459
5	0.0372	0.0327	0.0505	0.0459
10	0.0330	0.0293	0.0491	0.0459
50	0.0258	0.0241	0.0485	0.0459
∞	0.0217	0.0217	0.0484	0.0459

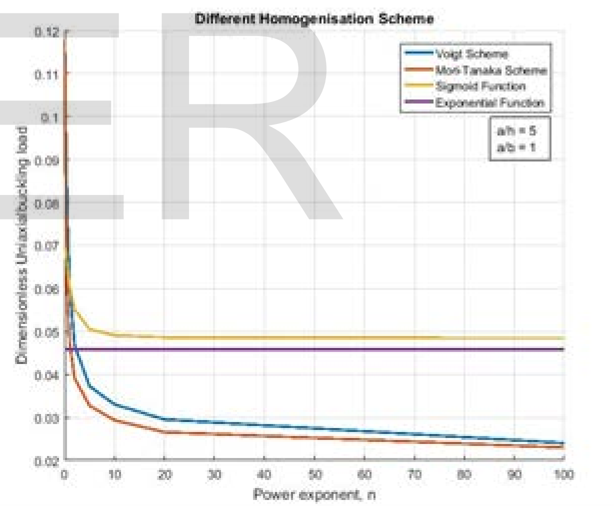

Fig.13. Variation of dimensionless uniaxial buckling load w.r.t 'n' values.

Table.8. dimensionless uniaxial buckling load for different aspect ratios 'a/b' for different homogenization schemes.

a/b	V-FGM	M-FGM	S-FGM	E-FGM
0.5	0.0195	0.0165	0.0228	0.0192
1	0.0467	0.0391	0.0552	0.0459
1.5	0.0842	0.0673	0.1035	0.0825
2	0.1148	0.0887	0.1459	0.1123
2.5	0.1360	0.1017	0.1771	0.1328
3	0.1514	0.1131	0.1997	0.1478

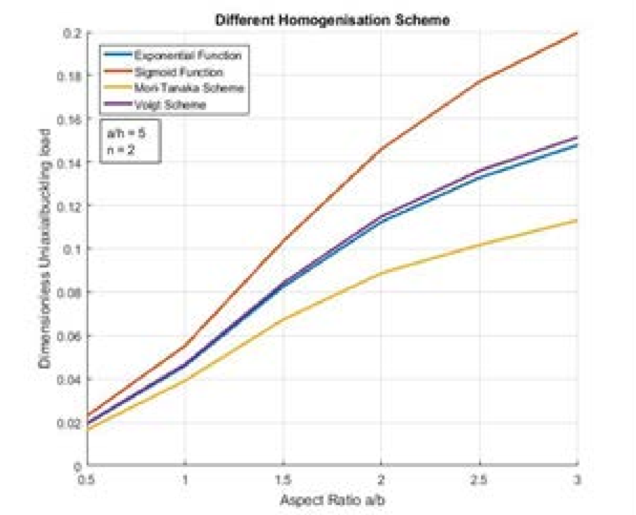

Fig.14. Variation of dimensionless uniaxial buckling load w.r.t 'a/b' values.

Table.9. dimensionless uniaxial buckling load for different side-thickness ratios 'a/h' for different homogenization schemes.

a/h	V-FGM	M-FGM	S-FGM	E-FGM
1	0.1883	0.1390	0.2518	0.1829
2	0.1360	0.1017	0.1788	0.1327
5	0.0467	0.0391	0.0552	0.0459
10	0.0134	0.0115	0.0155	0.0131
20	0.0035	0.0030	0.0040	0.0034

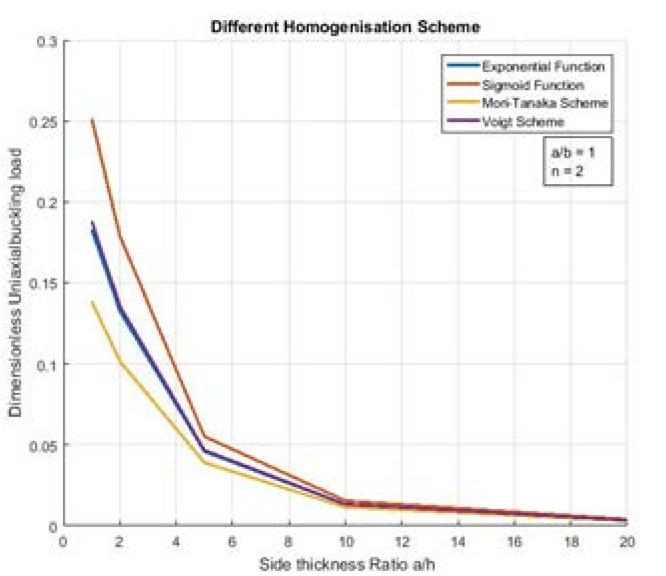

Fig.15. Variation of dimensionless uniaxial buckling load w.r.t 'a/h' values.

7.3 Biaxial Buckling Load

The buckling load obtained using Reddy's TSDT has been validated. The result is made dimension-less using following expression,

$$N = \frac{Ncr}{h * Ec}$$

where 'N' is the dimension-less natural frequency, 'Ncr' is the natural frequency, 'h' is the plate thickness and 'Ec' is the young's modulus of the ceramic material.

Table.10. dimensionless biaxial buckling load for different power exponent 'n' for different homogenization schemes.

n	V-FGM	M-FGM	S-FGM	E-FGM
0	0.0590	0.0590	0.0350	0.0229
0.5	0.0391	0.0298	0.0327	0.0229
1	0.0303	0.0236	0.0303	0.0229
5	0.0186	0.0164	0.0252	0.0229
10	0.0165	0.0147	0.0245	0.0229
50	0.0129	0.0120	0.0242	0.0229
∞	0.0109	0.0109	0.0242	0.0229

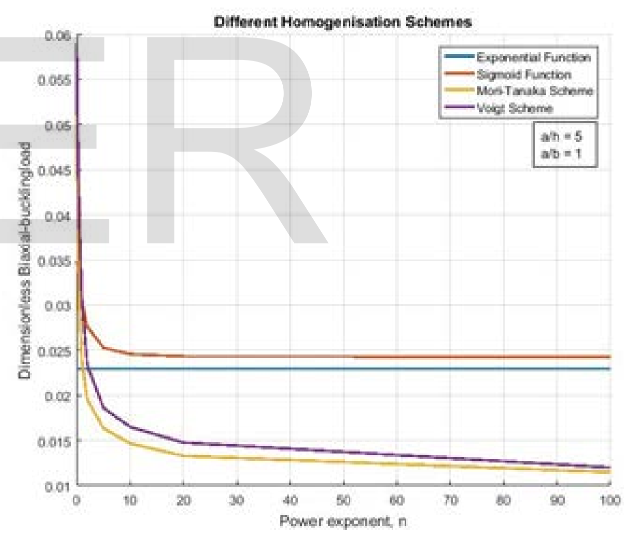

Fig.16. Variation of dimensionless biaxial buckling load w.r.t 'n' values.

Table.11. dimensionless biaxial buckling load for different aspect ratios 'a/b' for different homogenization schemes.

a/b	V-FGM	M-FGM	S-FGM	E-FGM
0.5	0.0156	0.0132	0.0182	0.0153
1	0.0233	0.0195	0.0276	0.0229
1.5	0.0344	0.0283	0.0412	0.0337
2	0.0467	0.0377	0.0568	0.0458
2.5	0.0589	0.0468	0.0729	0.0577
3	0.0701	0.0549	0.0881	0.0687

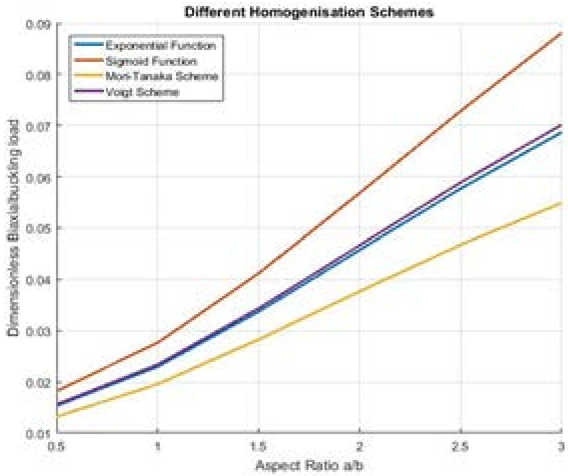

Fig.17. Variation of dimensionless biaxial buckling load w.r.t 'a/b' values.

Table.12. dimensionless biaxial buckling load for different side-thickness ratios 'a/h' for different homogenization schemes.

a/h	V-FGM	M-FGM	S-FGM	E-FGM
1	0.1227	0.0910	0.1632	0.1196
2	0.0781	0.0774	0.0991	0.0764
5	0.0233	0.0195	0.0276	0.0229
10	0.0086	0.0057	0.0077	0.0066
20	0.0017	0.0015	0.0020	0.0017

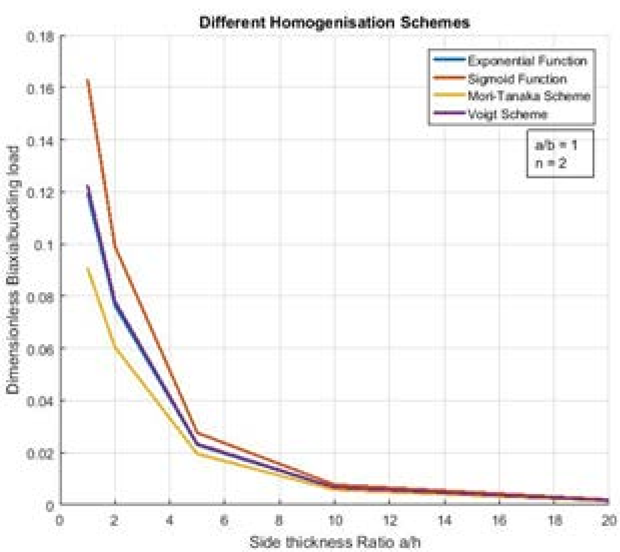

Fig.18. Variation of dimensionless biaxial buckling load w.r.t 'a/h' values.

8 CONCLUSION AND FUTURE SCOPE

The following points can be concluded from the conducted comparative studies.

1. Among the various homogenization schemes used Exponential function is independent of power exponent 'n' values and hence E-FGM model gives a constant value for both natural frequencies and buckling loads when plotted for different 'n' values.
2. Both natural frequency and buckling loads decreased with increasing 'n' value as the metallic nature is increasing, that makes the plate more flexible.
3. Both natural frequency and buckling loads increased as 'a/b' ratio increased, as the plate would become smaller and smaller thus less flexible.
4. Both natural frequency and buckling loads decreased with increase in 'a/h' ratio as the plate is behaving like a thin plate.
5. The results for both natural frequency and buckling loads calculated using different homogenization schemes tends to converge as the 'a/h' ratio increases as the plate is behaving like thin plates.
6. Sigmoid function showed small variation w.r.t 'n' value than other methods. This is clear from the plots for both natural frequency and buckling loads that S-FGM model shows a gradual variation than E-FGM and V-FGM model.
7. E-FGM and V-FGM models give similar results for both natural frequency and buckling loads when plotted for different 'a/h' and 'a/b' ratios.
8. S-FGM gives the upper-bound results while Mori-Tanaka gives the lower-bound.
9. V-FGM and M-FGM gives same results for both n= 0 (fully ceramic) and n= ∞ (fully metallic) for both natural frequency and buckling loads.
10. V-FGM and M-FGM gives results closer to the 3d solution with Mori-Tanaka Scheme being the better method however being a bit complex compared to Voigt scheme.

Some areas for future study includes following points:

1. The present study was conducted for only one boundary condition, i.e. all around simply supported, this can be extended for other boundary conditions also.
2. Thermal environment may be imposed in addition to the mechanical loading.

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